

# Is the Bell Experiment Really Loophole-Free?

## Abstract

We found some inappropriate assumptions in CHSH's thesis "Proposed Experiment to Test Local Hidden-Variable Theories" and Horne's doctoral thesis "Experimental Consequences of Local Hidden Variable Theories". In these theses, the polarizer was falsely selected as a measuring tool for experiments without considering the nonlinearity of its transmittance. According to our calculation, using non-linear tools for measurement, the experimental data will always violate the CHSH inequality. We further analyzed several existing CHSH experiments and found that they face the same problem.

## INTRODUCTION

In 1935, Einstein, Podolsky, and Rosen presented the EPR paradox, convinced that the theory of quantum mechanics is incomplete<sup>[1]</sup>. However, Bohr held that the measurement behavior inevitably influences the measurement result, and the paradox does not exist<sup>[2]</sup>.

In 1951, Bohm introduced the hidden variable theory, presuming that there are deeper reasons behind the randomness of microscopic particles which should be explained by hidden variables<sup>[3]</sup>.

In 1964, John S. Bell proposed the famous Bell inequality,  $1 + P(b, c) \geq |P(a, b) - P(a, c)|$ , in "On the Einstein Podolsky Rosen Paradox" <sup>[4]</sup>.

In 1969, Clauser, Horne, Shimony, and Holt (CHSH) published the thesis "Proposed Experiment to Test Local Hidden-Variable Theories", and derived the CHSH inequality,  $|E(a, b) - E(a, b') + E(a', b) + E(a', b')| \leq 2$ , which can be used for practical testing<sup>[5]</sup>.

In the following 50 years, a number of teams conducted experiments using the method proposed by CHSH. Some well-known teams are:

1972, Stuart J. Freedman and John F. Clauser<sup>[6]</sup>

1981, Alain Aspect, Philippe Grangier, and Gérard Roger<sup>[7]</sup>

2015, Ronald Hanson, Bas Hensen et al. <sup>[8]</sup>

2016, The Big Bell Test Collaboration<sup>[9]</sup>

These experiments, without exception, declared that CHSH inequality and Bell inequality were violated, and the experimental data supported the prediction of quantum theory. Furthermore, these experiments successively closed detection loophole, locality loophole, and free-will loophole. The hidden variable theory was seemingly eliminated.

Thus, is the Bell experiment really loophole-free as claimed? How do we explain the spooky quantum entanglement phenomenon?

To facilitate the understanding, some basic definitions must be

established:

$\theta$ : angle between the photon polarization direction and the polarizer transmission axis (polarization angle)

$\epsilon$ : transmittance of the polarizer

$\epsilon_M$ : transmittance when the light polarization direction is parallel ( $\theta = 0^\circ$ ) to the polarizer transmission axis (parallel transmittance)

$\epsilon_m$ : transmittance when the light polarization direction is perpendicular ( $\theta = 90^\circ$ ) to the polarizer transmission axis (cross transmittance)

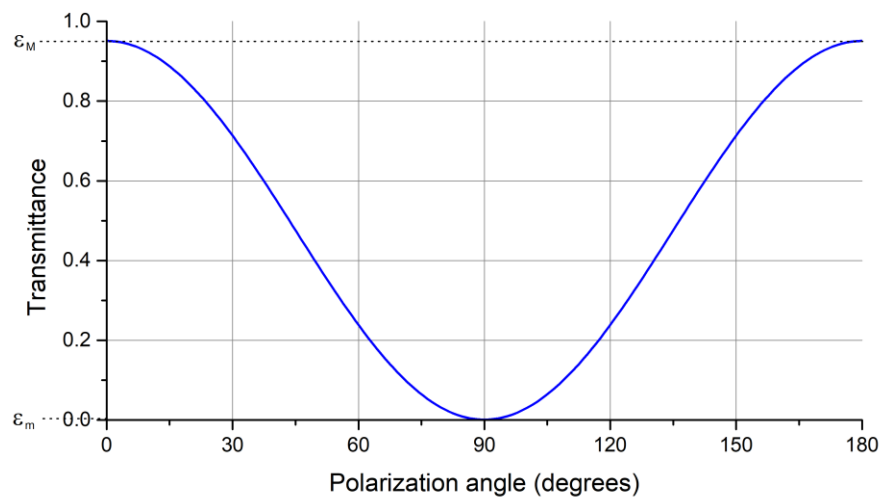


FIG. 1. Transmittance curve for non-ideal polarizer.

(See Fig. 1) According to the Malus' law, for plane polarized light with an intensity of  $I_0$ , after passing through the polarizer, the intensity of the transmitted light is  $I = I_0(\cos\theta)^2$ . Therefore, the transmittance of an ideal polarizer  $\epsilon = I/I_0 = (\cos\theta)^2$ . When  $\theta \in [0^\circ, 90^\circ]$ ,  $\epsilon = f(\theta)$  is a curve, rather than a straight line (note: for simplicity, the straight line, curve, linear and non-linear mentioned later all refer to the interval of  $\theta \in [0^\circ, 90^\circ]$ , which will not be repeated). The relation between the transmittance  $\epsilon$  of a non-ideal polarizer and the angle  $\theta$  is much more complicated, so that we cannot use a uniform function to describe the transmittance curves for all non-ideal polarizers. However, we can conclude from experiments that the transmittance of the non-ideal polarizer  $\epsilon = f(\theta)$  remains non-linear. Statistically, the probability  $P$  of a single photon passing through the polarizer equals the transmittance  $\epsilon$  of the polarizer. Therefore,  $P = f(\theta)$  is also non-linear.

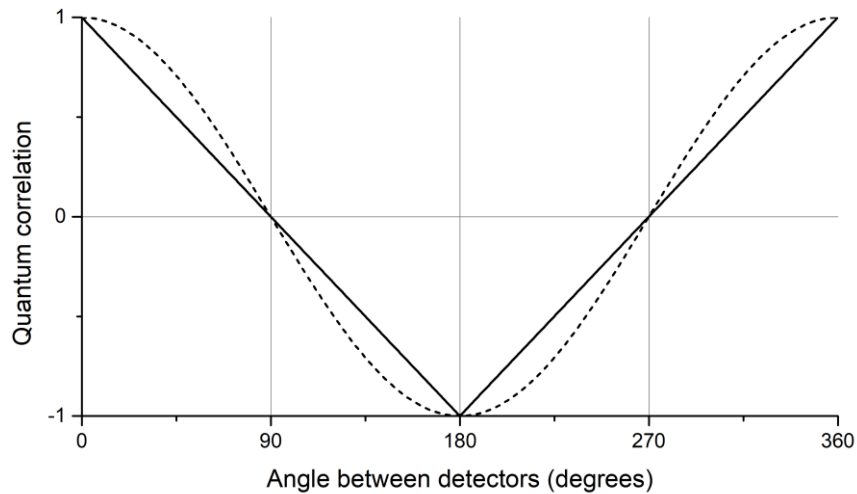


FIG. 2. Prediction results of hidden variable theory (solid line) and those of quantum theory (dotted curve).

(See Fig. 2) Bell experiment principle: a parent particle with zero angular momentum decays into a pair of spin 1/2 particles in singlet state and moving in the opposite directions. At two distant locations, measure the spin directions of two particles in three dimensions. Then calculate the correlation of these twin particle pairs according to classical probability theory. If the hidden variable theory is correct, the Bell inequality is true; if the quantum theory is correct, the Bell inequality is not true.

The Bell experiment is only theoretically feasible. Limited by measuring means, we cannot experiment directly with the method assumed by Bell. Subsequently, CHSH published a thesis in which the CHSH inequality was derived from the Bell inequality. Inspired by the experiment of Kocher and Commins (KC)<sup>[10]</sup>, CHSH proposed to generate twin photon pairs by atomic cascade emission, and use the polarization direction of the photon to replace the spin direction of the particle. The experiment by KC demonstrated that the twin photon pairs emitted in cascade of calcium are related when passing through two polarizers placed parallel or perpendicular, and that the polarization direction of photon corresponds to the spin direction of the particle.

Everything seems to be all right. Then what is the truth?

Let us consider an example first:

Alice and Bob test the telepathy between them by tossing coins. Normally, Alice and Bob both have a 50% chance of getting either heads or tails. Therefore, the probability that they get the same side is  $0.5 \times 0.5 + 0.5 \times 0.5 = 0.5$ . Now someone adds weight to the heads of the two coins in secret. In such a case, when Alice and Bob toss the coins, the probability of getting heads is 40%, and that of getting tails is 60%. The probability for them to get the same side is  $0.4 \times 0.4 + 0.6 \times 0.6 = 0.52$ . Hence, the consistency between Alice and Bob has increased significantly. People mistakenly believe that there is telepathy between Alice and Bob.

Yes, it is the polarizer that causes the discrepancy. Theoretically, when the polarization angle  $\theta = 45^\circ$ , the probability of photons passing through the polarizer must be 50%. In fact, when  $\theta = 45^\circ$ , the transmittance of the non-ideal polarizer is not 50%. This increases the consistency of the twin photon pairs as they pass through the polarizer. The transmittance of the polarizer is non-linear, and so is the probability of photons passing through the polarizer. Therefore, the number of photons passing through the polarizer cannot form a strict proportional relation with the polarization direction of photons. CHSH mistakenly selected the non-linear polarizer as the measuring tool, making the statistical data obtained from the experiment accordingly inaccurate.

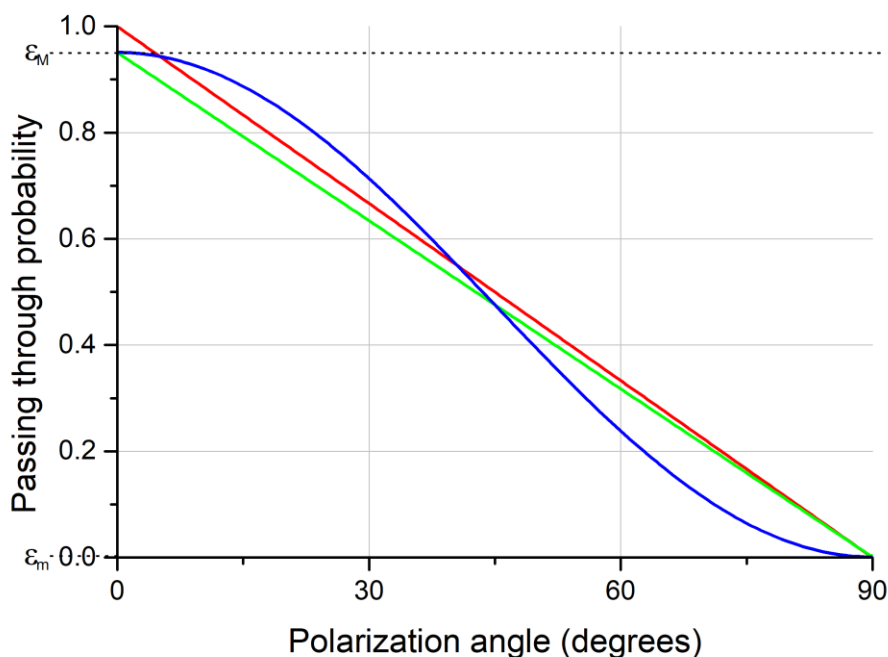


FIG. 3. The relation between the passing through probability  $P$  and the polarization angle  $\theta$ .

(See Fig. 3) The relation between the passing through probability  $P$  and the polarization angle  $\theta$  when photons pass through different measuring tools:

1. Ideal measuring tool required by the experiment: the straight line connecting coordinates (0, 1) and (90, 0) (red).
2. The polarizer assumed by CHSH: the straight line connecting coordinates (0,  $\epsilon_M$ ) and (90,  $\epsilon_m$ ) (green).
3. The actual polarizer: the uncertain curve connecting coordinates (0,  $\epsilon_M$ ) and (90,  $\epsilon_m$ ) (blue).

CHSH supposed that there is a detection efficiency error between the non-ideal polarizer and the ideal measuring tool, but the error can be eliminated by calculation to achieve the same effect as the ideal measuring tool. Nevertheless, the actual transmittance of the non-ideal polarizer is not the green straight line as assumed by CHSH, but the blue curve shown in the figure. Because the probability of photons passing through the polarizer is

non-linear, and the transmittance curves of polarizers with different wavelengths, different materials, and different batches are different, we cannot adjust it to the ideal measuring tool using the fixed calculation formula. Both Bell inequality and CHSH inequality were derived from the unitary linear hidden variable theory. Using a non-linear polarizer as the measuring tool obviously cannot meet the requirements of the experiment.

## PROBLEMS IN CHSH'S THESIS

Here, let us analyze the problems in CHSH's thesis "Proposed Experiment To Test Local Hidden-Variable Theories" in detail<sup>[5]</sup>:

In principle entire measuring devices, each consisting of a filter followed by a detector, could be used for  $I_a$  and  $II_b$ , and the values  $\pm 1$  of  $A(a)$  and  $B(b)$  would denote detection or nondetection of the particles. Inequalities (1) would then apply directly to experimental counting rates. Unfortunately, if the particles are optical photons (as in the experiment proposed below) no practical tests of (1) can presently be performed in this way, because available photoelectric efficiencies are rather small. We shall therefore henceforth interpret  $A(a)=\pm 1$  and  $B(b)=\pm 1$  to mean emergence or nonemergence of the photons from the respective filters. Also the filters will be taken to be linear polarization filters, and  $a$  and  $b$  will represent their orientations. It will be convenient to introduce an exceptional value  $\infty$  of the parameter  $a$  (and likewise of  $b$ ) to represent the removal of a polarizer; clearly,  $A(\infty)$  and  $B(\infty)$  necessarily equal  $+1$ . Since  $P(a, b)$  is an emergence correlation function, in order to derive an experimental prediction from (1) an additional assumption<sup>9</sup> must be made: that if a pair of photons emerges from  $I_a, II_b$  the probability of their joint detection is independent of  $a$  and  $b$ . Then if the flux into  $I_a, II_b$  is a constant independent of  $a$  and  $b$ , the rate of coincidence detection  $R(a, b)$  will be proportional to  $w[A(a)_\pm, B(b)_\pm]$ , where  $w[A(a)_\pm, B(b)_\pm]$  is the probability that  $A(a)=\pm 1$  and  $B(b)=\pm 1$ . Letting  $R_0=R(\infty,$

FIG. 4. CHSH's thesis (P.881) assumed that the joint detection probability of the photons is independent of the orientations of the polarizers.

(See Fig. 4) To derive the experimental results, an additional assumption was made in CHSH's thesis: "that if a pair of photons emerges from  $I_a, II_b$  the probability of their joint detection is independent of  $a$  and  $b$ ." This is a strange assumption. If it is true, then that is also independent of the angle between  $a$  and  $b$ . Thus, it is meaningless to set the polarizers with different angles to conduct the CHSH experiment. KC's experiment demonstrated that twin photon pairs passing through two polarizers with an angle of  $0^\circ$  are positively correlated, whereas those passing through two polarizers with an angle of  $90^\circ$  are negatively correlated. This additional assumption is evidently wrong. The consistency of a pair of photons passing through the polarizers varies with the angle and transmittances of the polarizers, and the variation is non-linear. If the preconditions of the assumption are wrong, the subsequent derivation process and experimental results are accordingly not convincing.

Proposed experiment.—A decisive test can be obtained by modifying the KC experiment to include observations at two appropriate relative orientations of the polarizers, and also with one and then the other removed. For realizable apparatus, quantum mechanics predicts violation of inequality (2b).

Define  $\epsilon_M^i$  as the efficiency of the polarizer  $i$  ( $i=I, II$ ) for light polarized parallel to the polarizer axis and  $\epsilon_m^i$  as that for light perpendicularly polarized. Consider a point source and filter-detector assemblies, each of which gathers the photons emitted into a cone of half-angle  $\theta$ . Then for a  $J=0 \rightarrow J=1 \rightarrow J=0$  electric-dipole cascade (0-1-0) the quantum mechanical predictions for the counting rates are<sup>10</sup>

$$R(\varphi)/R_0 = \frac{1}{4}(\epsilon_M^{II} + \epsilon_m^{II})(\epsilon_M^{II} + \epsilon_m^{II}) + \frac{1}{4}(\epsilon_M^I - \epsilon_m^I)(\epsilon_M^{II} - \epsilon_m^{II})F_1(\theta) \cos 2\varphi, \\ R_1/R_0 = \frac{1}{2}(\epsilon_M^I + \epsilon_m^I), \quad R_2/R_0 = \frac{1}{2}(\epsilon_M^{II} + \epsilon_m^{II}). \quad (3)$$

Here  $\varphi$  is the angle between the polarizer axes,

$$F_1(\theta) = 2G_1^2(\theta)[G_2^2(\theta) + \frac{1}{2}G_3^2(\theta)]^{-1},$$

FIG. 5. Formulas in CHSH's thesis (P.882), the detailed derivation was adopted from Horne's doctoral thesis.

(See Fig. 5) CHSH's thesis defined  $\epsilon_M$  as the efficiency of the polarizer for light polarized parallel to the polarizer axis and  $\epsilon_m$  as that for light perpendicularly polarized; the  $R(\varphi)/R_0$  formula was derived from this. According to the definitions in the thesis, we believe that  $\epsilon_M$  and  $\epsilon_m$  are the parallel transmittance and cross transmittance of the polarizer, respectively. Because the transmittance of the polarizer is non-linear, its transmittance curve cannot be determined only by  $\epsilon_M$  and  $\epsilon_m$ . Therefore, the  $R(\varphi)/R_0$  formula in CHSH's thesis does not hold true. According to the definitions in this thesis, either  $R_1/R_0$  or  $R_2/R_0$  is the single transmittance of the polarizer on one side with the polarizer on the other side removed, which is the average value of the transmittance curve. It must be the area of the transmittance curve of a polarizer  $\epsilon = f(\theta)$  in the interval  $\theta \in [0^\circ, 90^\circ]$  divided by the interval length. The value  $1/2(\epsilon_M + \epsilon_m)$  in the thesis has no geometrical meaning for uncertain curves.

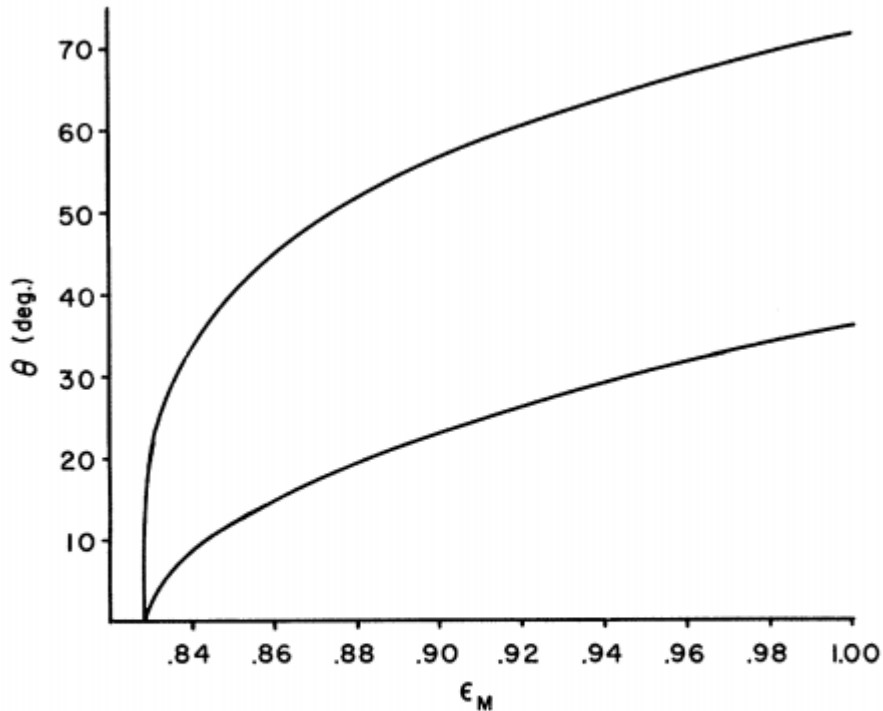


FIG. 1. Upper limits on detector half-angle  $\theta$  as a function of polarizer efficiency  $\epsilon_M$ . To test for hidden-variable theories, the experiment must be performed in the region below the appropriate curve—the upper curve for a 0-1-0 cascade, the lower for a 0-1-1.

FIG. 6. Upper limit of polarizer efficiency  $\epsilon_m$  calculated in CHSH's thesis (P.883).

(See Fig. 6) CHSH's thesis claimed that the experimental requirements can be met as long as  $\epsilon_m$  is small enough, and  $\epsilon_M$  is larger than a certain upper limit. Then, we can assume that we have a polarizer whose transmittance curve is very steep and almost vertical. The transmittance  $\epsilon$  tends to 1 ( $\epsilon_M \rightarrow 1$ ) in the infinitesimal interval near  $\theta = 0^\circ$  and rapidly approaches 0 ( $\epsilon_m \rightarrow 0$ ) in the interval  $\theta \in (0^\circ, 90^\circ]$ . We refer to this polarizer as the black hole polarizer. This is completely in line with the requirements for the polarizer in this thesis; however, hardly any photons can pass through the black hole polarizer. It can be concluded that the experimental data is related to the complete polarizer transmittance curve, not only to  $\epsilon_M$  and  $\epsilon_m$ .

## PROBLEMS IN HORNE'S THESIS

The thesis published by CHSH in the Physical Review Letters was simplified without mentioning many details. The detailed derivation process was adopted from Horne's doctoral thesis. Let us take a look at Horne's doctoral thesis "Experimental Consequences of Local Hidden Variable Theories"<sup>[11]</sup>.

For both cascades, it is shown that the quantum mechanical and the DLHV predictions are in sufficient disagreement for an actual experimental test to be possible. Restrictions on detector solid angle and polarizer efficiency, necessary to obtain a decisive experiment, are explicitly given. The Einstein, Podolsky, and Rosen argument for the existence of a DLHV theory is reviewed.

FIG. 7. Horne's thesis (P.ii) claimed that non-ideal polarizers have efficiency problems.

(See Fig. 7) Horne believed that non-ideal polarizers have efficiency errors. However, the transmittance  $\epsilon$  and angle  $\theta$  of actual polarizers are non-linear, thus we cannot use the efficiency coefficient to adjust the curve of the non-ideal polarizer to the straight line of the ideal measuring tool. The difference between the non-ideal polarizer and the ideal measuring tool must not be considered as the efficiency error.

there is the third channel of reflection (or absorption). We could still interpret +1 and -1 as in the ideal experiment if we assume that the loss of a photon into the third channel is independent of its state  $\lambda$  and the orientation of the polarizer,  $a$ . With such a random depletion of the total ensemble, the emerging pairs of photons would still constitute a faithful sample of the initial statistical ensemble characterized by  $\rho(\lambda)$ . Then the experimental determination of  $P_{ij}(a,b)$  would be as in the ideal case except that  $N_e$  instead of  $N_0$  would be used in (4.10) for correct normalization.

FIG. 8. Horne's thesis (P.40) claimed that photons will be lost in the non-ideal polarizer due to reflection or absorption.

(See Fig. 8) Horne's thesis claimed that there is a third channel in the non-ideal polarizer due to reflection or absorption, and photons will be lost by entering it. It assumed that the lost photon is independent of the state  $\lambda$  and the orientation of the polarizer, such that ideal statistical samples can be obtained by subtracting lost photons from the total number of photons. However, the thesis did not provide sufficient evidence to prove that the probability of reflection or absorption when photons pass through the polarizer at different polarization angles is the same. Hence, this assumption is not rigorous enough.



The above assumption is not in general implied by a DLHV theory. However, the DLHV theory does imply that the state  $\lambda$  determines any binary decision at the apparatus  $a$ . Thus we redefine the +1 and -1 results:  $A(\lambda, a) = +1$  [ $B(\lambda, b) = +1$ ] denotes that the photon is transmitted in the ordinary ray;  $A(\lambda, a) = -1$  [ $B(\lambda, b) = -1$ ] denotes that it is not transmitted in the ordinary ray (i.e., it is either in the extraordinary ray or in the third channel). Since now the required observation on each photon is whether it is transmitted in the ordinary ray of the polarizer or not, observations of the extraordinary ray are irrelevant. Thus we can employ the more common calcite polarizer that transmits one ray without deviation and deflects the other into a black absorption coating (Glan type polarizers).

FIG. 9. Horne's thesis (P.40) argued that observations of the extraordinary ray are irrelevant.


(See Fig. 9) Horne's thesis claimed that according to the hidden variable theory, which channel a photon enters is determined by state  $\lambda$ , regardless of the measuring equipment. It redefined +1 indicates that the photon is transmitted in the ordinary ray, whereas -1 indicates that the photon is transmitted in the extraordinary ray or in the third channel. The thesis held that in the experiment, it is only required to measure the ordinary light passing through the polarizer, and the measurement of the extraordinary light is irrelevant. In fact, the state  $\lambda$  only dictates the polarization direction of photons. The channel which a photon enters is not only determined by the polarization direction, but also by the polarizer in the measuring equipment. The photons entering the third channel are also counted as -1, which is equivalent to weighting the heads of the coins, thus changing the consistency of the photons.

It may appear that the assumption of constant detector efficiency can be established experimentally by measuring detection rates when a controlled flux of photons of known polarization impinges on each detector. From the standpoint of hidden variable theories, however, these measurements are irrelevant, since the distribution of the hidden variables when the fluxes are thus controlled is almost certain to be different from the  $\rho(\lambda)$  governing the ensemble of cascade photons. In view of the difficulty of an experimental check, considerable effort has been made to show that this assumption is unnecessary. Simple DLHV

FIG. 10. Horne's thesis (P.43) assumed that the detector efficiency is a constant.

(See Fig. 10) Horne's thesis assumed that the detector efficiency is a constant, which can be demonstrated experimentally. However, the measurements are irrelevant due to the difficulty of the experiment. Nevertheless, the transmittance of the polarizer is actually non-linear, such that we cannot evaluate an uncertain curve with the efficiency constant.

Now we give the operators (efficiency matrices) corresponding to each linear polarizer. In a basis of linear polarizations along  $x_1$  and  $y_1$  in the coordinates of photon 1, the most general linear polarizer with axis along  $x_1$  is described by an efficiency matrix

$$\epsilon^{(1)} = \begin{pmatrix} \epsilon_M^1 & 0 \\ 0 & \epsilon_m^1 \end{pmatrix},$$


i.e.,  $\epsilon_M^1$  is the probability of transmitting an  $x_1$  linearly polarized photon, and  $\epsilon_m^1$  is the probability of transmitting a  $y_1$  linearly polarized photon (leakage). In the ideal

FIG. 11. Polarizer efficiency matrix defined in Horne's thesis (P.52).

(See Fig. 11) Horne's thesis defined a polarizer efficiency matrix. This matrix only defined the upper limit  $\epsilon_M$  and the lower limit  $\epsilon_m$ , which could not accurately express the non-linear transmittance curve  $\epsilon$  of the polarizer. In this thesis, the linear polarizer was mistakenly assumed to be the polarizer with the

linear transmittance. In fact, the linear polarizer should be referred to as the plane polarizer. Its transmittance remains non-linear. Because we cannot use a uniform function to describe the transmittance curves for all non-ideal polarizers, a uniform  $R(\varphi)/R_0$  formula cannot be derived.

## GEDANKEN EXPERIMENT

Then, what is the difference between experimenting with linear and non-linear measuring tools? How is spooky quantum entanglement generated?

For convenient calculation, we must assume an ideal experimental environment. The polarization direction of the twin photon pair generated each time in the experiment is random, but the polarization directions of the two photons in the twin photon pair are the same, and photons will not be entangled and deflected due to the measurement.

Let  $E(a, b)$  be the normalized correlation function of the twin photons passing through the measuring instruments  $a$  and  $b$ .

$$E(a, b) = \frac{P(a_+, b_+) + P(a_-, b_-) - P(a_+, b_-) - P(a_-, b_+)}{P(a_+, b_+) + P(a_-, b_-) + P(a_+, b_-) + P(a_-, b_+)} \quad (1)$$

Here  $P(a_+, b_+)$  is the probability for photons passing through both  $a$  and  $b$ ;  $P(a_-, b_-)$  is the probability for photons passing through neither  $a$  nor  $b$ ;  $P(a_+, b_-)$  is the probability for photons passing through  $a$  but not  $b$ ;  $P(a_-, b_+)$  is the probability for photons passing through  $b$  but not  $a$ .

Define  $P(a_+, \infty)$  as the probability for photons passing through  $a$  with instrument  $b$  removed, and  $P(\infty, b_+)$  as the probability for photons passing through  $b$  with instrument  $a$  removed. Under ideal conditions,

$$\begin{aligned} P(a_+, b_+) + P(a_-, b_-) + P(a_+, b_-) + P(a_-, b_+) &= 1 \\ P(a_+, \infty) &= P(a_+, b_+) + P(a_+, b_-) = 1/2 \\ P(\infty, b_+) &= P(a_+, b_+) + P(a_-, b_+) = 1/2 \end{aligned} \quad (2)$$

We can get a simpler formula from Eqs. (1) and (2):

$$\begin{aligned} E(a, b) &= P(a_+, b_+) + P(a_-, b_-) - P(a_+, b_-) - P(a_-, b_+) \\ &= 4 \times P(a_+, b_+) - 2 \times P(a_+, \infty) - 2 \times P(\infty, b_+) + 1 \\ &= -1 + 4 \times P(a_+, b_+) \end{aligned} \quad (3)$$

Because a large number of photons pass through measuring instruments at random angles,  $E(a, b)$  is related to the angle  $\varphi$  between  $a$  and  $b$ , and unrelated to the specific orientation of  $a$  and  $b$ . Hence,

$$E(a, b) = f(\varphi) = -1 + 4 \times P(\varphi_+) , \quad (4)$$

where  $\varphi = |a - b|$  and  $\varphi \leq 90^\circ$ .

According to the CHSH inequality

$$|E(a, b) - E(a, b') + E(a', b) + E(a', b')| \leq 2 , \quad (5)$$

take  $a = 0^\circ$ ,  $b = 22.5^\circ$ ,  $a' = 45^\circ$ ,  $b' = 67.5^\circ$ .

Assuming that the measuring instrument is an ideal measuring tool, then for a linear ideal measurement tool,  $P(\varphi_+)$  is required to be  $1/2 \times (1 - 2 \times \varphi/\pi)$ :

$$\begin{aligned}
f(\varphi) &= -1 + 2 \times (1 - 2 \times \varphi/\pi) \\
E(a, b) &= f(22.5^\circ) = 0.5 \\
E(a, b') &= f(67.5^\circ) = -0.5 \\
E(a', b) &= f(22.5^\circ) = 0.5 \\
E(a', b') &= f(22.5^\circ) = 0.5 \\
|E(a, b) - E(a, b') + E(a', b) + E(a', b')| &= 2
\end{aligned} \tag{6}$$

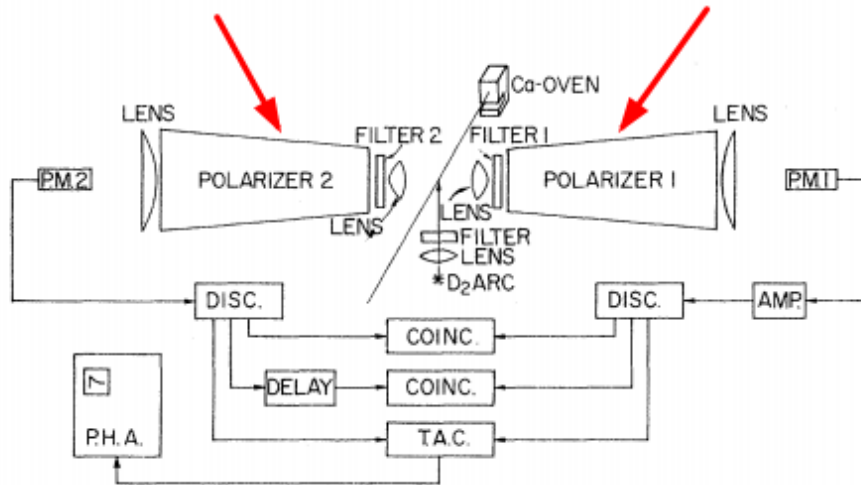
Assuming that the measuring instrument is an ideal polarizer, then for a non-linear ideal polarizer,  $P(\varphi_+)$  is approximately equal to  $1/2 \times (\cos \varphi)^2$ :

$$\begin{aligned}
f(\varphi) &\approx -1 + 2 \times (\cos \varphi)^2 \\
E(a, b) &= f(22.5^\circ) \approx 0.7071 \\
E(a, b') &= f(67.5^\circ) \approx -0.7071 \\
E(a', b) &= f(22.5^\circ) \approx 0.7071 \\
E(a', b') &= f(22.5^\circ) \approx 0.7071 \\
|E(a, b) - E(a, b') + E(a', b) + E(a', b')| &\approx 2.8284
\end{aligned} \tag{7}$$

Based on the above calculation, we made a virtual measurement with the ideal measuring tool and the ideal polarizer without the intervention of quantum theory, and obtained the result of the CHSH inequality. Evidently, the results obtained with linear and non-linear measuring tools are opposite. When the linear ideal measuring tool is used, the calculation results satisfy the CHSH inequality; however, when a non-linear ideal polarizer is used, the calculation results violate the CHSH inequality. Nevertheless, this violation is not caused by quantum entanglement, but by the non-linear measuring tool.

## PROBLEMS IN CHSH EXPERIMENTS

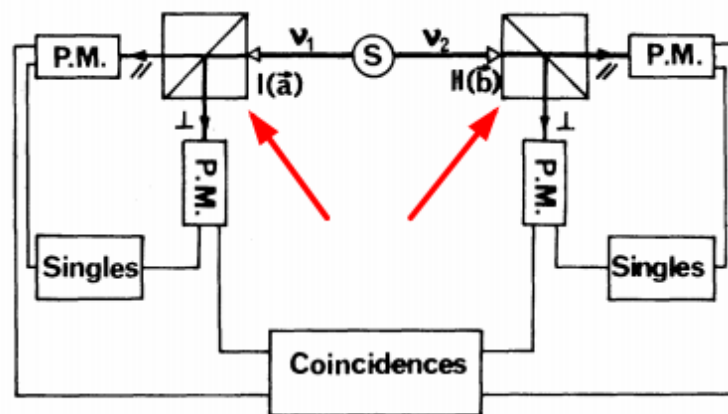
Finally, let us analyze the problems of the CHSH experiments mentioned at the beginning.



**FIG. 1. Schematic diagram of apparatus and associated electronics. Scalers (not shown) monitored the out-**

FIG. 12. Diagram of experimental setup of Stuart J. Freedman and John F. Clauser (P.939).

(See Fig. 12) In 1972, Stuart J. Freedman and John F. Clauser carried out the CHSH experiment and published a thesis<sup>[6]</sup>. The pile of glass-plates polarizer was employed in the experiment. The parallel transmittance and cross transmittance of the polarizer were measured. Nonetheless, no complete polarizer transmittance curve was provided in the thesis, thus it did not prove that the transmittance of the polarizer is linear to the polarization angle. Further, only the photons entering the +1 channel were detected in the experiment, without detecting those entering the -1 channel. This experimental method inevitably led to data deviation.



**FIG. 2. Experimental setup. Two polarimeters I and II, in orientations  $\vec{a}$  and  $\vec{b}$ , perform true dichotomic measurements of linear polarization on photons  $\nu_1$  and  $\nu_2$ . Each polarimeter is rotatable around the axis of the incident beam. The counting electronics monitors the singles and the coincidences.**

FIG. 13. Diagram of experimental setup of Alain Aspect, Philippe Grangier, and Gérard

(See Fig. 13) In 1981, Alain Aspect, Philippe Grangier, and Gérard Roger conducted the CHSH experiment and published a thesis<sup>[7]</sup>. In the experiment, the dual-channel polarizer made of two prisms was used as the measuring tool. The transmission and reflection coefficients of the dual-channel polarizer were provided. However, no complete transmittance/reflectance curve for the polarizer was provided in this thesis; it also did not prove that the transmittance/reflectance of the polarizer is linear to the polarization angle. Therefore, the experimental results are insufficient to prove that the CHSH inequality is violated.

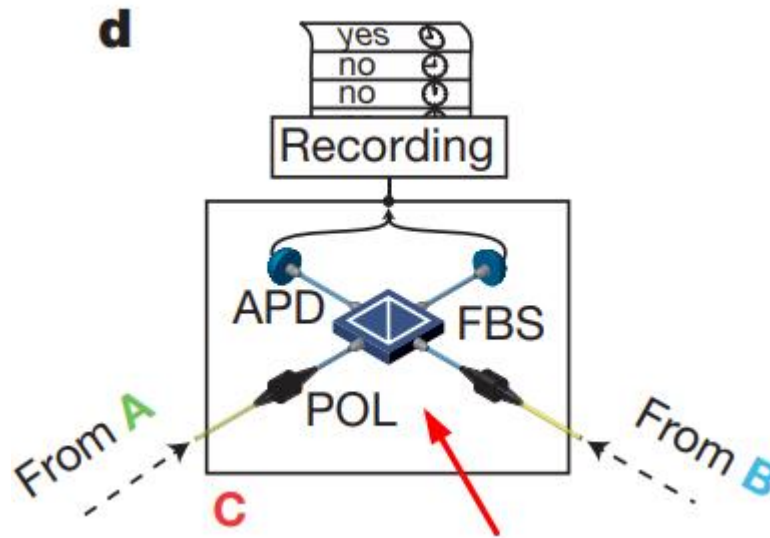


FIG. 14. Diagram of experimental setup of Ronald Hanson and Bas Hensen et al. (P.683).

(See Fig. 14) In 2015, Ronald Hanson and Bas Hensen et al. conducted the CHSH experiment<sup>[8]</sup>. Various optical instruments such as optical fiber, dichroic mirror, wave plate, polarizer, beam splitter, etc. were used in the experiment. However, no evidence was provided in the thesis to show that the probability of photons passing through these optical instruments is linear to the polarization angle. Therefore, the experimental results cannot prove that the CHSH inequality is violated.

**Table 1 | Experiments carried out as part of the BBT, ordered by longitude, from east to west**

Experiment	Lead Institution	Location	Entangled system	Rate (bps)	Inequality	Result	Stat. sig.
(1)	Griffith University	Brisbane, Australia	Photon polarization	4	$S_{16} \leq 0.511$	$S_{16} = 0.965 \pm 0.008$	$57\sigma$
(2)	University of Queensland & EQUUS	Brisbane, Australia	Photon polarization	3	$ S  \leq 2$	$S_{AB} = 2.75 \pm 0.05$ $S_{BC} = 2.79 \pm 0.05$	$15\sigma$ $16\sigma$
(3)	USTC	Shanghai, China	Photon polarization	$10^3$	PRBLG <sup>30</sup>	$I_0 = 0.10 \pm 0.05$	N/A
(4)	IQOQI	Vienna, Austria	Photon polarization	$1.61 \times 10^3$	$ S  \leq 2$	$S_{HRN} = 2.639 \pm 0.008$ $S_{QRN} = 2.643 \pm 0.006$	$81\sigma$ $116\sigma$
(5)	Sapienza	Rome, Italy	Photon polarization	0.62	$B \leq 1$	$B = 1.225 \pm 0.007$	$32\sigma$
(6)	LMU	Munich, Germany	Photon-atom	1.7	$ S  \leq 2$	$S_{HRN} = 2.427 \pm 0.0223$ $S_{QRN} = 2.413 \pm 0.0223$	$19\sigma$ $18.5\sigma$
(7)	ETHZ	Zurich, Switzerland	Transmon qubit	$3 \times 10^3$	$ S  \leq 2$	$S = 2.3066 \pm 0.0012$	$P < 10^{-99}$
(8)	INPHYNI	Nice, France	Photon time bin	$2 \times 10^3$	$ S  \leq 2$	$S = 2.431 \pm 0.003$	$140\sigma$
(9)	ICFO	Barcelona, Spain	Photon-atom ensemble	125	$ S  \leq 2$	$S = 2.29 \pm 0.10$	$2.9\sigma$
(10)	ICFO	Barcelona, Spain	Photon multi-frequency bin	20	$ S  \leq 2$	$S = 2.25 \pm 0.08$	$3.1\sigma$
(11)	CITEDEF	Buenos Aires, Argentina	Photon polarization	1.02	$ S  \leq 2$	$S = 2.55 \pm 0.07$	$7.8\sigma$
(12)	UdeC	Concepción, Chile	Photon time bin	$5.2 \times 10^4$	$ S  \leq 2$	$S = 2.43 \pm 0.02$	$20\sigma$
(13)	NIST	Boulder, USA	Photon polarization	$10^5$	$K \leq 0$	$K = (1.65 \pm 0.20) \times 10^{-4}$	$8.7\sigma$

FIG. 15. List of entanglement systems used in the Big Bell test (P.215).

(See Fig. 15) The Big Bell test was conducted in 2016<sup>[9]</sup>. It was a multi-site and multi-system collaborative experiment. Multiple quantum entanglement systems were employed in the experiment, and most were in the mode of photon polarization. If the polarizer or similar optical instruments were used in the experiment, the influence of the non-linear transmittance of the polarizer must be considered. We are unable to assess the results of this experiment due to the lack of further details.

## SIMPLE PROOF

A simple proof of CHSH experimental method error:

1. According to the Malus' law  $I = I_0(\cos\theta)^2$ , the transmittance of an ideal polarizer  $\epsilon = I/I_0 = (\cos\theta)^2$ , is a curve rather than a straight line when in the interval  $\theta \in [0^\circ, 90^\circ]$ .
2. For a non-ideal polarizer, the transmittance curve  $\epsilon$  cannot be determined only by  $\epsilon_M$  and  $\epsilon_m$ .
3. In the CHSH experiment, photons pass through the non-ideal polarizer at random polarization angles  $\theta$ . The experimental data is related to the complete polarizer transmittance curve  $\epsilon$ , not only to  $\epsilon_M$  and  $\epsilon_m$ .

Therefore, all CHSH experimental data that do not provide the complete polarizer transmittance curve are invalid.

Theoretically, the Bell experiment was supposed to measure the upward and downward spin direction of particles. In fact, the CHSH experiment calculated the number of photons that passed or did not pass through the polarizer. The CHSH experiment mistakenly assumed these two as the ratio data, and there is no convincing experimental data or theoretical basis was provided to show that the probability of photons passing through the polarizer is linear to the polarization angle. A wrong experimental method led to wrong results, and the experimental data cannot effectively prove that Bell inequality and CHSH inequality are violated.

## CONCLUSIONS

Based on the above analysis, the following conclusions can be drawn:

1. Both Bell inequality and CHSH inequality are significant discoveries; however, the data obtained with a non-linear measuring tool cannot prove that the inequalities are violated.
2. Correct experimental results can only be obtained by experimenting with linear measuring tools or correcting the experimental data with the polarizer transmittance curve.
3. There is still no conclusive evidence on whether the hidden variable theory or the quantum theory holds true at present.

## REFERENCES

1. A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete?, *Physical Review* 47, 777 (1935).
2. N. Bohr, Can quantum-mechanical description of physical reality be considered complete?, *Physical Review* 48, 696 (1935).
3. D. Bohm, A suggested interpretation of the quantum theory in terms of 'hidden' variables. i and ii, *Physical Review* 85, 166 and 180 (1952).
4. J. S. Bell, On the einstein podolsky rosen paradox, *Physics* 1, 195 (1964).
5. J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed experiment to test local hidden-variable theories, *Physical Review Letters* 23, 880 (1969).
6. S. J. Freedman and J. F. Clauser, Experimental test of local hidden-variable theories, *Physical Review Letters* 28, 938 (1972).
7. A. Aspect, P. Grangier, and G. Roger, Experimental realization of einstein-podolsky-rosen-bohm gedankenexperiment: a new violation of bell's inequalities, *Physical Review Letters* 49, 91 (1982).
8. B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau, and R. Hanson, Loophole-free bell inequality violation using electron spins separated by 1.3 kilometres. *Nature* 526, 682 (2015).
9. The BIG Bell Test Collaboration, Challenging local realism with human choices, *Nature* 557, 212 (2018).
10. C. A. Kocher and E. D. Commins, Polarization correlation of photons emitted in an atomic cascade, *Physical Review Letters* 18, 575 (1967).
11. M. A. Horne, Experimental consequences of local hidden variable theories, Ph. D. thesis (1970).